Mixed Topic FRQ Classwork

1.

| $x$ | 1 | 1.1 | 1.2 | 1.3 | 1.4 |
|---|---|---|---|---|---|
| $f'(x)$ | 8 | 10 | 12 | 13 | 14.5 |

The function $f$ is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of $f'$, the derivative of $f$, are given for selected values of $x$ in the table above.

(a) Write an equation for the line tangent to the graph of $f$ at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\,dx$. Use the approximation for $\int_1^{1.4} f'(x)\,dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

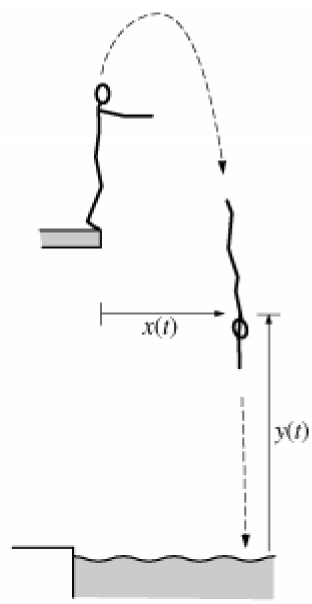
(d) Write the second-degree Taylor polynomial for $f$ about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

2.

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time $t$ seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \le t \le A$, where $A$ is the time that the diver's shoulders enter the water.



Note: Figure not drawn to scale.

(a) Find the maximum vertical distance from the water surface to the diver's shoulders.

(b) Find $A$, the time that the diver's shoulders enter the water.

(c) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(d) Find the angle $\theta$, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

3.

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

(b) At the point $(-1, 2)$, the value of $\frac{d^2y}{dx^2}$ is $-12$. Find the second-degree Taylor polynomial for $f$ about $x = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

4.

The Maclaurin series for $e^x$ is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function $f$ is defined by $f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \neq 1$ and $f(1) = 1$. The function $f$ has derivatives of all orders at $x = 1$.

(a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.

(b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for $f$ about $x = 1$.

(c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).

(d) Use the Taylor series for $f$ about $x = 1$ to determine whether the graph of $f$ has any points of inflection.